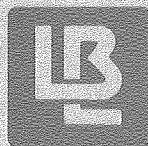


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Lawrence Berkeley Laboratory

UNIVERSITY OF CALIFORNIA

Physics, Computer Science & Mathematics Division

To be submitted to the American Journal of Physics

"DERIVATION" OF THE DE BROGLIE RELATION FROM THE
DOPPLER EFFECT

Frank S. Crawford

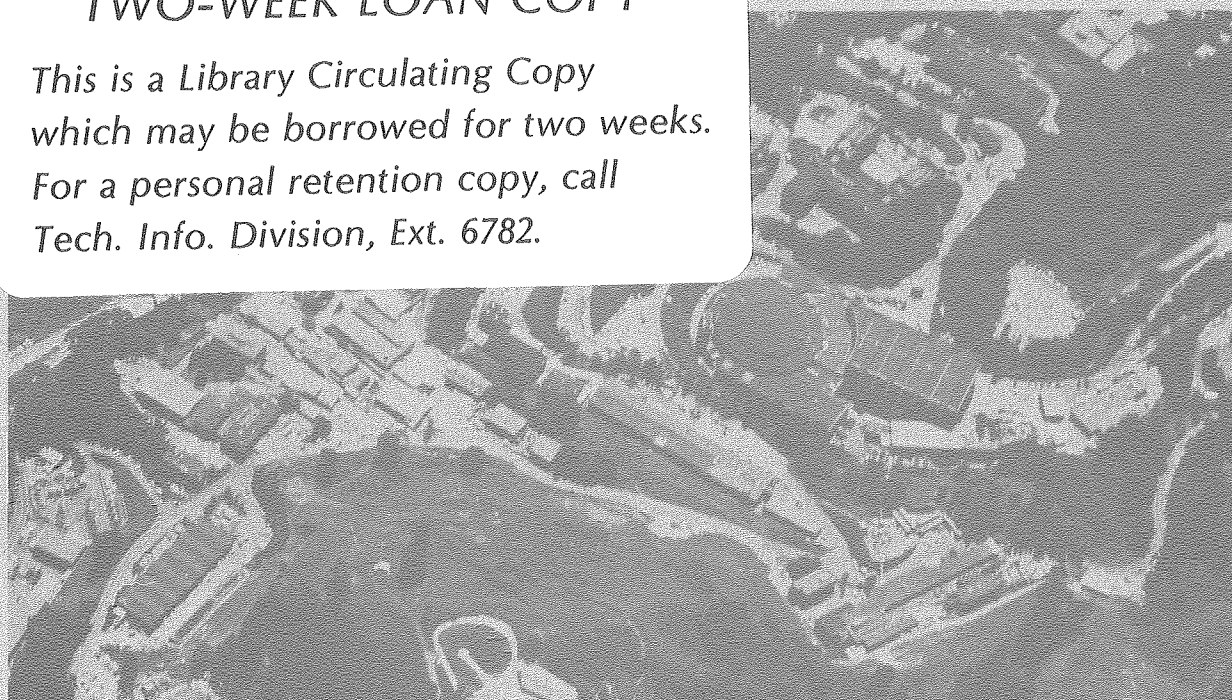
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"DERIVATION" OF THE DE BROGLIE RELATION FROM THE DOPPLER EFFECT

Frank S. Crawford
Physics Department
and
Lawrence Berkeley Laboratory
University of California
Berkeley, CA 94720

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In a note entitled "Simple illustration of the equivalence of a wave and of a particle description for light", T. Ferbel¹ shows that if light is reflected at normal incidence from a massive mirror receding from the light source with low mirror speed $V_0 \ll c$ then there are two entirely different ways to show that the wave length λ is increased after reflection by an amount $d\lambda$ given by

$$d\lambda/\lambda = 2V_0/v, \quad (1)$$

where $v=c$ is the speed of light. The first way is to assume light is a wave. Then the usual derivation of the Doppler effect gives Eq.(1). (The unfamiliar factor of 2 in Eq.(1) can be thought of in terms of images: if the mirror moves with speed V_0 the image of a stationary light source moves with speed $2V_0$.) The second way is to assume that light is a particle having zero rest mass, and thus having energy E momentum p and velocity v related by

$$E = cp, \quad v=c. \quad (2)$$

With the additional assumption that these particles have wave properties and obey the de Broglie relation

$$\lambda p = h, \quad (3)$$

where h is Planck's constant, and with the assumption of energy and momentum conservation in the elastic collision with the wall (a "Compton" collision), Ferbel shows that one again obtains Eq.(1). The particle picture is in that sense equivalent to the wave picture.

In this note we generalize the argument from photons to all particles-- non-relativistic electrons, relativistic electrons, photons, etc-- and also turn the argument around: We assume that all these particles have wave properties and hence must exhibit a Doppler shift given by Eq.(1) when they reflect from a massive slowly receding mirror. We then show that all these particles must satisfy the de Broglie relation (3), with h an unknown constant to be determined by interference experiments to measure λ for particles of known momentum.

Before we can generalize to other particles besides photons we must remove an ambiguity in Eq.(1): does v represent the phase velocity v_ϕ or does it instead represent the group velocity v_g ? Or perhaps neither? For light in vacuum we have $v_\phi = v_g = c$ and the ambiguity does not arise; but for material particles the phase and group velocities are different. Therefore we must remove the ambiguity from Eq.(1).

Consider a stationary observer and a wave source retreating from the observer with velocity v_s , with $v_s \ll v_\phi$. During the period T_0 in which the source emits one wave the source travels a distance $v_s T_0$ away from the observer and the "leading edge" of the wave travels a distance $v_\phi T_0$ towards the observer, where v_ϕ is the phase velocity at the new "stretched" wavelength λ , which is longer than the original wavelength λ_0 . The time T for this wave to pass the stationary observer is

$$T = (v_s T_0 + v_\phi T_0) / v_\phi = T_0 [1 + (v_s / v_\phi)]. \quad (4)$$

For the shift in period T , and in angular frequency $\omega = 2\pi/T$, we therefore have

$$d\omega/\omega = -dT/T = -v_s/v_\phi. \quad (5)$$

To obtain the Doppler shift in wavelength we could return to Eq.(4) and use the fact that $v_\phi T = \lambda$. But there is an easier way.

The phase and group velocities are given by

$$v_\phi = \omega/k, \quad v_g = d\omega/dk, \quad (6)$$

with $k = 2\pi/\lambda$. Therefore we can simply write down

$$-d\lambda/\lambda = dk/k = (dk/d\omega)(d\omega/\omega)(\omega/k) = (v_\phi/v_g)(d\omega/\omega). \quad (7)$$

Combining Eqs.(7) and (5) then gives

$$d\lambda/\lambda = v_s/v_g, \quad (8)$$

which resolves the ambiguity. For the case of interest, the moving source is the image of a stationary source in a mirror moving with velocity V_0 , so that $v_s = 2V_0$, and Eq.(8) becomes

$$d\lambda/\lambda = 2V_0/v_g. \quad (9)$$

We are now ready to consider elastic scattering of a particle from the slowly moving wall. We consider first the "opposite extreme" from a photon--a nonrelativistic electron.

Let a massive mirror be traveling to the right with very low velocity $V=V_0$. A nonrelativistic electron of rest mass m_0 and velocity v is traveling to the right, catching up with the mirror. (We need $v>V_0$.) Let the electron velocity after the elastic collision be v' , with v' negative if the electron is moving to the left after the collision and positive if it is still moving to the right. An elementary calculation using conservation of energy and momentum yields the well-known result that the collision does not change the magnitude of the relative velocity of the electron and the mirror but just reverses its sign. (The sign reverses because they are moving together before the collision and apart afterwards.) Assume the mirror is so massive that its velocity is unchanged by the collision. Equating the relative velocity "before" to its negative "after" gives

$$v-V_0 = -(v'-V_0), \quad (10)$$

i.e.,

$$v'=2V_0-v. \quad (11)$$

We see that if v is greater than $2V_0$ then v' is negative and the particle bounces back to the left. We are only interested in the case where v is very large compared with $2V_0$. In that case the speed $|v'|$ after the collision equals $-v'$. Then the shift in particle speed, which we call dv , is given by

$$dv = |v'| - v = -v'-v=-2V_0. \quad (12)$$

Now divide Eq.(12) by the initial speed v to get

$$dv/v = -2V_0/v. \quad (13)$$

Notice that Eq.(13) closely resembles Eq.(9) except for the important minus sign.

Now assume that these nonrelativistic electrons have both a particle velocity v and a wavelength λ . We then assume that both Eqs.(13) and (9) hold, and we assume v_g in (9) equals the particle velocity v in (13). Adding Eqs.(9) and (13) then gives the differential relation $d[\ln(\lambda v)]=0$, which integrates to give $\lambda v = \text{constant}$. Multiplying this constant by the rest mass m_0 gives

$$\lambda m_0 v = \text{constant}. \quad (14)$$

Eq.(14) is equivalent to Eq.(3) for a nonrelativistic particle having $p=m_0 v$. The numerical value of the constant must be determined by other experiments.

Now consider the more general case of a relativistic electron catching up with the same mirror and undergoing an elastic collision. We again assume the mirror is so massive that its velocity is unchanged by the collision. For a relativistic particle the connection between total energy E , momentum p , rest mass m_0 , and velocity v can be written

$$E^2 = c^2 p^2 + (m_0 c^2)^2, \quad (15)$$

$$v = c^2 p / E. \quad (16)$$

(For the special case that the rest mass m_0 is zero, Eqs.(15) and (16) reduce to Eq.(2) and we then have the case considered by Ferbel.) Before the collision the electron momentum has magnitude p and the electron is traveling to the right catching up with the mirror. After the collision the electron is traveling to the left with momentum of magnitude p' . Energy and momentum conservation give

$$E + \frac{1}{2} M V^2 = E' + \frac{1}{2} M V'^2, \quad (17)$$

$$p + M V = -p' + M V', \quad (18)$$

where M , V , and V' refer to the massive mirror.

Rearranging terms these become

$$E - E' = \frac{1}{2} M (V'^2 - V^2), \quad (17')$$

$$p + p' = M (V' - V). \quad (18')$$

Divide (17') by (18'). Then take $V'=V = V_0$. Also take $p'+p=2p$; i.e., p' is only slightly less than p . Call $E'-E=dE$. Then (17') and (18') give

$$dE/p = -2V_0. \quad (19)$$

Note that differentiation of Eq.(15) gives $E dE = c^2 p dp$, i.e.,

$$dE/p = c^2 dp/E = (dp/p)(c^2 p/E) = v dp/p, \quad (20)$$

where we used v from Eq.(16) in the last step. Combining (20) and (19) gives

$$dp/p = -2V_0/v. \quad (21)$$

Eq.(21) is the relativistic generalization of Eq.(13). Now assume that relativistic electrons have wave properties and obey Eq.(9) as well as (21). Adding (9) and (21) then gives the differential relation $d[\ln(\lambda p)] = 0$, which integrates to give

$$\lambda p = \text{constant}. \quad (22)$$

Eq.(22) is the relativistic generalization of Eq.(14). It holds for all particles--slow electrons, relativistic electrons, photons, etc.

An interesting parallel to the ambiguity in Eq.(1) [$v=v_\phi?$, $v_g?$] is the following ambiguity: Suppose we know the results of the photoelectric effect and can therefore write for the photon energy $E=hf$, where f is the frequency. For light, $f=c/\lambda$, and for particles of zero rest mass, $E=cp$. Therefore we can immediately write down that for photons we have $\lambda=h/p$, and we also have the completely equivalent formula $\lambda=hc/E$. If we now want to guess the relation for material particles, should we guess $\lambda=h/p$? Or should it be $\lambda=hc/E$, with perhaps $E=p^2/2m$ for non-relativistic particles, and given by Eq.(15) for relativistic particles? We have shown how, by using the correct Doppler shift, we can unambiguously decide that the correct relation must be $\lambda=h/p$, not hc/E , for material particles. It is interesting, and we leave it as an exercise for the student, that if we had, incorrectly, taken the Doppler shift in wavelength to be $d\lambda/\lambda = v_s/v_\phi$, then we would have

obtained the incorrect result $\lambda=hc/E$, where E is the kinetic energy for a nonrelativistic particle, or the total energy for a relativistic particle.

To summarize: once we believe that light, nonrelativistic electrons, relativistic electrons, etc., must all have both wave and particle properties then by considering elastic reflection from a slowly moving massive mirror we can "derive" the de Broglie relation as expressed by Eq.(22). Of course there are other ways, and that is not the way de Broglie did it.²

ACKNOWLEDGEMENTS

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1. T. Ferbel, Am. J. Phys. 48, 675 (August, 1980).
2. For an excellent discussion see E. H. Wichmann, Berkeley Physics Course Vol.4 (McGraw-Hill, 1967), Chap.5.

